HIGH Q COMPLEX POLE REALIZATION BY RC ACTIVE NETWORK SYNTHESIS

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UNITED STATES
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THESIS

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12ND P2238 (1-59)

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Submitted in partial fulfillment of
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IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

The realization of a transfer impedance with a single complex pole pair through RC active synthesis has led to applications in low pass circuits. One active source properly loaded will have the same percent change in its equivalent elements as the passive resistances and capacitances which complete the circuit.

The technique of adding additional active sources to obtain high Q complex pole pairs for bandpass applications is derived. Methods of loading the several active sources to restrict their variations are given. Limitations of the synthesis for very high Q's are discussed and some interesting aspects of the final circuitry pointed out.

I wish to thank Professor G. R. Giet of the U. S. Naval Postgraduate School for interesting me in advanced circuit theory through his excellent courses; Dr. Louis Weinberg of Hughes Aircraft Company who received me into his section, aided and advised me, and introduced me to modern synthesis methods.

Finally, I wish to credit Dr. I. M. Horowitz of Hughes, whose work is the foundation of this paper, since without his advice and encouragement, I should hardly have completed it.

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1. Introduction.

The advent of the transistor as a compact, reliable, low power device in the early 1950's encouraged designers to re-examine the possibilities of active RC structures as filter devices, particularly in low frequency applications.

Historically, ideal feedback amplifiers with RC elements in the feedback loop date from the late 1930's (1). Such an amplifier represented in figure 1 will give a transfer function:

$$\frac{E_{out}}{E_{in}} = \frac{G}{1 + GH}$$

Rewritten:

$$= \frac{G\,D(s)}{D(s) + G\,N(s)}$$

where $H = \dfrac{N(s)}{D(s)}$

If G is simply a gain constant the solution of the equation,

$$1 + K\frac{N(s)}{D(s)} = 0$$

determines the characteristic roots of the system. To produce a transfer function which is frequency selective the RC feedback circuit must have complex zeros.

RC structures which give complex zeros may be obtained by Dasher's or Guillemin's methods (2). A commonly used structure is a bridged or twin T which has the disadvantage that precise balance is required for proper operation. Further any drift in the passive elements is magnified by the resulting unbalance of the T.

The concept of negative impedance converters (NIC) advanced and applied by Linvill (3,4) makes it possible to realize any RLC transfer function through active RC circuits.

Beginning with the two port of figure 2 and partitioning the structure, the transfer impedance of the whole may be written in terms of the parts:

$$z_{21} = \frac{z_{21a}\, z_{21b}}{z_{22a} + z_{11b}}$$

By introducing a NIC between the two parts, which has as its chain matrix:

$$\begin{bmatrix} -k & 0 \\ 0 & 1 \end{bmatrix}$$

the resultant function would be:

$$z_{21}' = \frac{z_{21a}\, z_{21b}}{z_{22a} - k z_{11b}}$$

For simplicity take k as one, and the roots of

$$z_{22a} - z_{11b} = 0$$

determine the characteristic response of the network. Stable NIC are constructed from multiple transistor arrays, and within certain frequency ranges and for certain restricted impedance levels, k will remain within practical tolerances.

The difficulty of arriving at a characteristic equation by subtracting polynomials has been pointed out (5), and the attendant requirements for most stable NIC are a disadvantage of this approach to filter realization.

The cascaded RL-RC approach as set forth by Horowitz (6) offers some distinct advantages. The structure is simple, and no exact

subtraction problem appears. Further, it is possible in a straight-forward manner to desensitize the overall character of the filter to changes in active parameters.

The use of active RL-RC cascaded structures for low pass filter applications has been demonstrated (5, 7) and it is the aim of this paper to illustrate a further extension of this synthesis to bandpass applications of Q's five and greater.

The difficulty which constitutes the problem at hand is that desensitizing the filter is accomplished by degenerate loading of the active source. To obtain complex poles close to the jw axis however, high gain sources are required.

The requirement for a high gain source with small variation in equivalent parameters is then a contradiction which is not satisfied by the single transistor RL-RC cascade.

Multiple configurations of sources will provide the gain required; they need only be constrained to also satisfy the desired parameter stability.

2. RL-RC active synthesis of a single complex pole pair.

A general expression for a transfer impedance with a single complex pole pair is:

$$Z_{21} = \frac{K(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

From the general equations in hybrid parameters for a two port, it is readily seen that the transfer function Z_{21} has for its poles, the zeros of h_{22}. If the output admittance with the input open circuited is written;

$$Y_{OUT} = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s + a}$$

realization of Y_{out} then will also produce the desired transfer impedance poles if the current gain is merely some constant.

As an initial step, a shunt capacitor of unit magnitude is removed from the two port; the remaining admittance would be:

$$Y_{OUT} - s = Y'_{OUT} = \frac{s\left[2\zeta\omega_n - a\right] + \omega_n^2}{s + a}$$

For this expression to be RL or RC $2\zeta\omega_n \geq a$ and a is positive (8).

Since a transistor is to be the active structure, the general shape of the remaining parts can be postulated as in figure 3. The output admittance in terms of this model is:

$$Y'_{out} = G_3 + \frac{1 + g Z_1}{R_2 + Z_1}$$

Subtracting G_3 from the algebraic expression for Y'_{out} leaves:

$$Y'_{out} - G_3 = \frac{1}{Z} = \frac{s\left[2\zeta\omega_n - a - G_3\right] + \left[\omega_n^2 - a \times G_3\right]}{s + a}$$

If the bracket expressions are positive, the remainder is still RL or RC. Z_1 may now be given in terms of the rest of the circuit:

$$Z_1 = \frac{Z - R_2}{1 - gZ}$$

If Z is RC, i.e. $a > \dfrac{\omega_n^2 - a \times G_3}{2\zeta\omega_n - a - G_3}$, the realization of the total circuit is simple.

The problem is: for a given Z which is RL, find R_2 and g so that Z_1 is RC (5).

Figure 4 is a sketch of an RL impedance in the complex frequency plane as a function of σ. The essential realizability condition for an RC impedance which assists in the final determination of R_2 and g is (9): poles and zeros alternate on the real axis with the least critical frequency yielding a pole.

From the expression for Z_1 in terms of the reduced transistor model, the zeros of Z_1 are given by;

$$Z = R_2$$

and similarly, the poles of Z_1 are the roots of:

$$Z = \frac{1}{g}$$

In order to satisfy the realizability condition above, in figure 5 R_2 must be $\geq$ Z_∞, and g^{-1} $\leq$ Z_0. This fixes the location of the poles and zeros of Z_1 under these conditions. It can be seen that this configuration of critical points is RC.

The expression $g^{-1} \leq Z_0$ may be written as an equality:

$$\gamma g = \frac{\omega_n^2}{a} \cdot b_1$$

γ is defined as a product of the transistor sensitivity factors defined below.

3. Determination of Q.

Since the active source is an intrinsic part of the total structure which gives the transfer impedance desired, there is a direct relationship between the elements of the transistor equivalent with its loading, and the position of the pole pair.

Referring to figure 6, the passive conductances are defined:

$$G_1 = G_1' + G_1''$$

$$G_2 = G_2' + G_2''$$

$$G_3 = G_3' + G_3''$$

G' is the conductance associated with the active source itself, while G" refers to shunt conductances added due to external loading.

There are four elements associated with the active source regardless of which transistor model is used. Writing the four in three expressions which are the ratio of the transconductance to the individual conductances, three sensitivity factors are defined:

$$\mathcal{N}_1 = g/G_1, \qquad \mathcal{N}_2 = g/G_2, \qquad \mathcal{N}_3 = g/G_3 .$$

Each may itself be written in two expressions which show the difference between $\mathcal{N}$ due to the transistor alone, and the transistor under some loading condition:

$$\mathcal{N}_1' = \frac{g}{G_1'} \; ; \quad \mathcal{N}_1 = \frac{g}{G_1' + G_1''} = \frac{g}{G_1}$$

γ is now given as $\dfrac{\mathcal{N}_2 + \mathcal{N}_3 - \mathcal{N}_2 f^2}{\mathcal{N}_2 \mathcal{N}_3 f}$

The exact relationship between the sensitivity factors and the pole position is (5):

$$\frac{1}{\rho^2} = \frac{\mu_2 \mu_3}{\mu_2 + \mu_3}\left[\frac{1}{\mu_3} - \frac{1+\mu_1}{\mu_1+\mu_2}\right]$$

Since: $$Q = \frac{1}{2\rho}$$

the determination of Q from the sensitivity factors is clear.

4. System sensitivity to parameter variation.

Variation of a system output or transfer characteristic due to changes in any parameter of the system has been defined in several ways in the literature (10, 11). The definition used by Horowitz directly relates the change in a particular parameter to the resultant actual movement of the pole position (6):

$$S_K^{s_0} = \frac{d s_0}{d K / K}$$

Appendix I lists the $S_K^{s_0}$ for both the active and passive elements in the RL-RC cascade. The only assumption made is that $f \ll 1$. $\Delta K / K$ is a factor usually specified by the manufacturer of the particular transistor.

5. Q and system performance.

For bandpass application, a shift in pole position affects the filter characteristic by altering the bandwidth or changing the center frequency, perhaps both. It would appear sensible to expect the higher the Q considered, the more stringent the requirements for insensitivity to parameter drift.

Thus for a Q of ten at 1,000 r.p.s., a five percent change in center frequency would be unacceptable since the bandwidth is but 100 r.p.s. and the drift has placed the filter passband so that half of the desired response is rejected. A more practical limit then for a Q of ten would be a change in center frequency of one percent or less.

For a Q of 20 with the same center frequency, perhaps a quarter of one percent variation in frequency would be the limit. It makes little sense to obtain a highly selective system if there is no stability to go with it.

Referring to Appendix I and considering only the elements associated with the transistor, for Q's in the range ten and above, the shift in the real direction of the pole is small. At higher Q's it may be necessary to determine accurately the sum of the magnitude of all real shifts due to each parameter to be certain that the system will not oscillate under any condition.

Figure 7 depicts the root locus for the complex pole pair where movement from the initial position is due to parameter variation. Since the motion is nearly vertical for small ρ the bandwidth of the filter is essentially a constant and the unwanted change is in the center frequency.

Still disregarding the effect of the capacitors it can be seen that G_2 and g are the elements which contribute to this change, and it is these two which require desensitizing.

6. Procedure for desensitizing.

The simplest way to achieve insensitivity in the shunt conductances G_1' and G_3' is to parallel them with load and source elements which overshadow the active portion in the expression:

$$G = G' + G''$$

The price that is exacted is a loss of current gain at the input and a loss of voltage gain at the output.

To reduce effects of g variation it is necessary to change the impedance level of the active source. Since the T to P^i transformation gives;

$$g = \frac{\alpha r_c}{|Z|}$$

the determinant $|Z| = r_c\left[r_e + r_b(1-\alpha)\right] + r_b r_e.$

For usual values $r_b r_e << r_c$

$$|Z| = r_c r_e'$$

where: $r_e' = r_e + r_b(1-\alpha)$

gives:

$$g = \frac{\alpha}{r_e'}$$

To alter g then, loading either the emitter leg or base leg is possible, with the more efficient method being emitter loading. Since the expression r_e' appears in all the P^i parameters, a change in g by altering r_e' also changes the other conductances of the active source.

Appendix II contains the expressions which determine the relative effect on sensitivity of all the active parameters by altering r_e' by m, thus:

$$r_e' m = R_e' .$$

7. Optimum μ for high Q.

From the expression, $$\frac{1}{Q^2} = \frac{\mu_2 \mu_3}{\mu_2 + \mu_3}\left[\frac{1}{\mu_3} + \frac{1+\mu_1}{\mu_1 + \mu_2}\right]$$

it can be determined what the best relationship between μ's will lead to high Q. If it is assumed that the shunt conductances are all non-zero, then all the μ's will be finite. The common multiplier $\frac{\mu_2 \mu_3}{\mu_2 + \mu_3}$ should be as large as possible and will be for the condition $\mu_2 = \mu_3$. Since μ_3 will be large, the term $1/\mu_3$ may be ignored and some optimum value for $\frac{1+\mu_1}{\mu_1+\mu_2}$ sought.

$\frac{1+\mu_1}{\mu_1+\mu_2}$ may have a value between zero and one. Since both μ_2, μ_3 are determined by the generator and load impedances, they are not apt to be a great deal larger than μ_2. However μ_1 is more likely the smallest and $\mu_1 + \mu_2$ is approximately μ_2. The right hand side may be written:

$$\frac{1}{2}\mu_1$$

This results in: $$Q = \sqrt{\frac{\mu_1}{8}}$$

For a Q of ten and the proper μ's, μ_1 must be 800.

If G_2 is small in comparison with the other three elements of the transistor model, μ_1, μ_3 will limit the Q obtainable. It may be practical to use transistor stages as impedance changers to isolate the effect of generator and load impedance from Q determination. An approximate structure, figure 8, shows that:

$$h_{21} = g/r_1 = \mu_1 \quad ; \quad \mu_3 = g R_3$$

μ_1 is then the short circuit current gain, and μ_3 is equivalent to open circuit voltage gain. For high Q, a transistor

device with high gain and a transconductance in the proper sense is required.

8. The single transistor

The common emitter (CE) is the only configuration of the three most often used which satisfies the proper direction of g. H_{21} of a CE may be in the range 50 to 100 for usual types, thus a Q of three plus is possible. For higher Q it is obvious that multiple arrays are necessary.

9. The compound transistor

The compound or composite transistor yields a single structure with α in the range .999 (12). The current gain in a common emitter compound then could be 1,000 or more. The equivalent P^1 derived from figure 9 gives values:

$$G_1 \doteq \frac{(1-\alpha_1)(1-\alpha_2)}{r_{e_2}'}$$

$$G_2 \doteq \frac{(r_{e_1} + r_{b_2})(1-\alpha_2)}{r_{e_2}' \, r_{c_1}}$$

$$G_3 \doteq \frac{1}{r_{e_2} + r_{c_1}(1-\alpha_1)(1-\alpha_2)}$$

$$g \doteq \frac{1}{r_{e_2}'}$$

While N_1 is increased by the factor $\frac{1}{1-\alpha}$, N_3 is also decreased by essentially the same factor. When the pair is stabilized by loading the second emitter leg, N_3 decreases directly as m.

Other compound connections do not provide the proper sense for g so that for the added transistor, no gain over the low Q obtainable from a single CE is obtained.

10. The cascade.

The usual benefit of cascading amplifier is multipulcation of gain. Figure 10 depicts two P^i models in a simple cascade. Again disregarding generator or output load impedance restrictions, the equivalent single structure has for its elements:

$$G_1 = G_1 + \frac{G_2(g_1 - G_c)}{G_c'}$$

$$G_2 = \frac{G_2 G_2'}{G_c'}$$

$$G_3 = G_3' + \frac{G_2'(g_2 + G_c)}{G_c'}$$

$$g = \frac{g_2(G_2 - g_1) + G_2' g_1}{G_c'}$$

$G_c = G_3 + G_1' + G_2$ and $G_c' = G_c + G_2 + G_2'$

The expression for the transconductance of the two shows that in either the first or last P_i, g must be negative. Thus some combination of a CE with a common base (CB) or common collector (CC) is required.

If both G_2 and G_2' are small, the approximate expression for g

$$g_{CASCADE} = \frac{-g_1 g_2}{G_c}$$

$S_g^{g_{CASCADE}}$ considering G_c a constant and with similar transistors would be (11):

$$\frac{d\, g_c / g_c}{d\, g / g} = 2$$

For a transistor with variations from the mean value of its four active parameters, the percent change is Δ k/k x 100; k being identified

as the parameter of interest. In the cascade, the percent change of the equivalent g is then twice that of the single transistor.

Appendix III lists the P^i equivalents for the three common configurations. The CE and CB possess the requirement for the approximate g-cascade, namely:

$$G_2 << g$$

The conductances G_1 and G_3 for the pair may be approximated under the same conditions as for g-cascade.

$$G_{1\,CASCADE} = G_1 + \frac{G_2 g_1}{G_c}$$

$$G_{3\,CASCADE} = G_3' + \frac{G_2' g_2}{G_c}$$

For the term involving a negative g, $\frac{G_2 g}{G_c}$ must be at least a magnitude less than the conductance of the single transistor,

$$G_1 > \frac{G_2 g_1}{G_c} \qquad G_3' > \frac{G_2' g_2}{G_c}$$

to avoid any appreciable subtraction which would entail a more severe stability requirement.

To stabilize g-cascade and limit pole shift assume $g_1 \times g_2$ is decreased by m with G_c still a constant. $S_{g_c'}^{g_c}$ then is 1/m. It makes no difference whether g_1 or g_2 is decreased by emitter leg loading, the effect on the overall g is the same.

If, however, G_c is essentially equal to G_1' and the loading is such as to decrease G_1' by the same factor m, $S_{g_c'}^{g_c}$ is simply unity. It can

be seen for degenerate loading to be effective in stabilizing g-cascade. G_c may be either a constant determined by the interstage loading, or the loading is done in the first stage.

To illustrate the relation of the magnitudes of the component transistors to the final equivalent elements, the following CE-CB pair is worked out.

$$r_e = 32\,\Omega \qquad r_b = [illegible]00\,\Omega \qquad r_c = 2.5 \times 10^{6}\,\Omega$$

$$\alpha = .985 \qquad r_e' = 40\,\Omega$$

Let m = 50 for the second stage. The final P^1 has for its values:

$$G_{r_1} = 3.75 \times 10^{-4}$$

$$G_{r_2} = 6.4 \times 10^{-10}$$

$$G_{r_3} = 3.2 \times 10^{-7}$$

$$g = 2.43 \times 10^{-3}$$

G_L was chosen as 5×10^{-3} to swamp out G_1' in value. G_{r_1} is far too small to admit a Q of any size, but this cascade of two is still of interest. The equivalent G_1 and G_3 are just the conductances of the first and last transistor in the same order.

In considering this cascade, no provisions were made for driving generator or load impedance influencing the result. For generator impedances of the order of 1,000 ohms, the only model which would provide a fair Q would require a large transconductance, say g = 1. If two CE's were cascaded, G_L could be raised to approximately 8×10^{-5} and the equivalent g is now between 1/10 and 2/10. The direction of g-cascade is now opposite to that required for synthesis.

The next step is simply to add another CE to obtain the desired direction in g and again increase its magnitude.

11. The triple CE cascade.

From the discussion above, the triple CE while requiring another transistor removes the problem of isolating the generator from the filter proper, and is in fact no more complex than a pair with isolation would be. Further, with large values of transconductance for the entire structure, N_2 may not need be equal to N_3, providing the designer a greater latitude in choice of the output impedance of the filter.

There are a number of practical factors associated with the final solution to the problem of providing large N's. If possible, the fewest number of precise passive elements should be required by the active source. Since the system is to be used at low frequency, compensation in the cascade should be simple or more ideally, not necessary. Power source requirements should be reasonable. Finally, the question of d.c. operating point stability for the individual transistors must be considered.

As has been indicated, there is a choice of where to load for stability, i.e., in what stage. The input impedance of a CE is essentially $h_{21} \times R_e$. R_e is the sum of r_e and the stabilizing resistor in the emitter leg.

At ten cycles, a 50 uf capacitor has a reactance of magnitude about 300 ohms. If the input impedance of a coupled stage is 3,000 ohms or more, there is little need for compensation at ten cycles or above. If h_{21} is 50, R_e need only be 60 ohms, or for usual values of r_e', m = 2. Loading in stages two and three in a triple cascade will then eliminate any need for low frequency compensation.

To remove the effect of G_1' from consideration G_L must be the dominant term of G_c.

G_L is the parallel combination of the d.c. collector load and the base bias resistor of the succeeding stage, as shown in Figure 11. G_a can be written $1/r \times G_b$. Then:

$$G_L = \left[\frac{1}{r} + 1\right] G_b$$

and Δ k for $G_L = \left(\frac{1}{r} + 1\right) \Delta G_b$

For r large, the variation in G_L can be attributed to G_b.

G_b can be approximated as $\frac{10(1-\alpha)}{m r_e'}$. The d.c. sensitivity of a chosen operating point for a biased CE in Figure 12, is given by (13):

$$S_{DC} = \frac{dI_c}{dI_{C_0}} = \frac{R_a + R_s}{R_a + R_s(1-\alpha)}$$

For the circuit under consideration R_s is approximately $m \times r_e'$, then:

$$S_{DC} = \frac{m r_e' + \frac{m r_e'}{10(1-\alpha)}}{m r_e' + \frac{m r_e'}{10}} = \frac{\beta + 10}{11}$$

The power source requirement is a function of r, since in the two battery bias scheme:

$$E_{cc} = V_{CE} + I_c R_a$$

and G_a will then represent the designer's choice between power supply requirements and increasing the precision required for two additional passive elements.

To summarize the use of the triple CE cascade:

1. Q's of ten are possible.

2. The restriction on generator impedance has been set at 1,000 ohms or more.

3. Pole shift insensitivity commensurate with the Q.

4. Practical circuit factors are reasonable.

Appendix IV presents the design of the structure yielding a complex pole pair of Q equal to ten. The same values for the transistor as in the previous calculation in section ten are used.

12. Extensions and limitations of a triple cascade.

The design example points out a most useful factor in the final circuit. Q determination is solely a function of the active source with its dissipative elements. The active source and the resistances of generator and load fix the lateral position of the pole pair.

The vertical position of the poles which determines the center frequency is a function only of the capacitors. Thus having settled on a Q it is only necessary to change the values of the capacitors to change the center frequency to any new figure desired.

Operating point sensitivity to changes in I_{c_o} need not be considered very restrictive. R_b can be made larger without too much difficulty as long as the system is not handling signals which are on the verge of distorting.

The gain of the triple system is given by (6):

$$\left|\frac{E_{out}}{E_{in}}\right|_{jw_n} = \frac{\mu_2(1-\rho^2)}{2\rho} \times \frac{\frac{1}{\mu_1} - \frac{1}{\mu_1'}}{\mu_2 - 1}$$

Under the assumptions made, G_v (voltage gain at jw_n) is approximately:

$$G_v \doteq \frac{\mu_2}{2\rho}\left(\frac{1}{\mu_1} - \frac{1}{\mu_1'}\right) \doteq Q\mu_2\left(\frac{1}{\mu_1} - \frac{1}{\mu_1'}\right)$$

Since μ_1' is at least 100 times μ_1 in the design example (g-cascade was decreased that much):

$$G_v = Q\,\mu_2/\mu_1$$

The total gain is then 500 at center frequency and is not large enough to overdrive the last stage for small input signal. Thus the operating point may shift somewhat without any real resultant harm.

For m large then, g will not decrease if G_L keeps step. Thus in the example, if the generator impedance is increased to 5,000 ohms, a Q of 20 is possible.

Another interesting point is the use of several complex pole pair structures in a chain to increase rejection outside of the passband. The output impedance of the triple cascade is high relative to the generator impedances so far assumed. The gain requirements of succeeding filters is reduced since the first structure isolates the generator from later filters.

The basic limitation of the cascade is not the result of the use of several active sources. Indeed, as m becomes large, the variation in g-cascade can be reduced to those of G_L. The passive interstage elements then determine the ultimate insensitivity of g-cascade.

It has been concluded previously that G_L is determined essentially by G_b. The precise elements required to keep the total pole shift within tolerance include then the two capacitors, two interstage resistors, the generator and output load impedances, and the bridged resistance R_2.

As long as the assumption is made that passive elements are constants, m can be increased with G_c keeping step, and Q's 20 and above are obtainable. For very high Q however, the simple assumption of stable passive elements may be no longer warranted.

Thus limitations in the known precision of the passive elements enumerated and of course the attendant cost of very precise elements may indicate a limit beyond which other systems may be competitive.

13. Conclusions.

The solution of the problem of obtaining a high gain source with desired insensitivity hinges on the simple observation that in the cascade, while gain is multiplied, variations of the additional active elements contribute only as a sum. Three elements are necessary to obtain the desired gain, and fortunately a choice of configurations is possible to allow the synthesis requirements to be met.

Rather than add more transistors, the possibility exists of using a two loop system to provide positive feedback to obtain the high gain necessary for large Q, and a negative feedback loop to assure the degree of insensitivity required. No attempt has been made to carry out the analysis of various two loop systems within the bounds of the RL-RC cascade method, however, such work might lead to a more economical system both from the point of numbers of elements required and the number of precise elements.

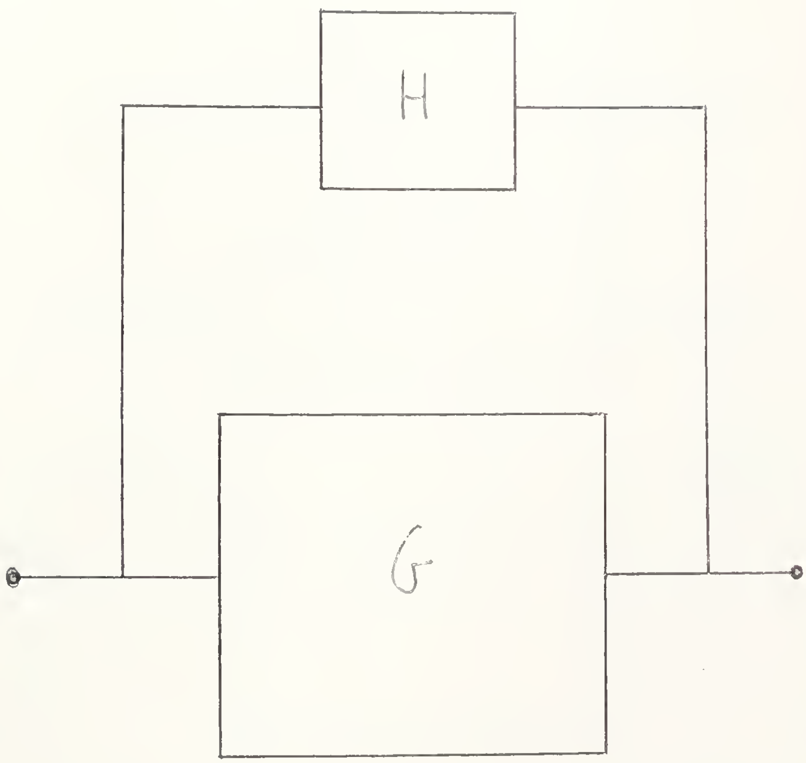


Figure 1

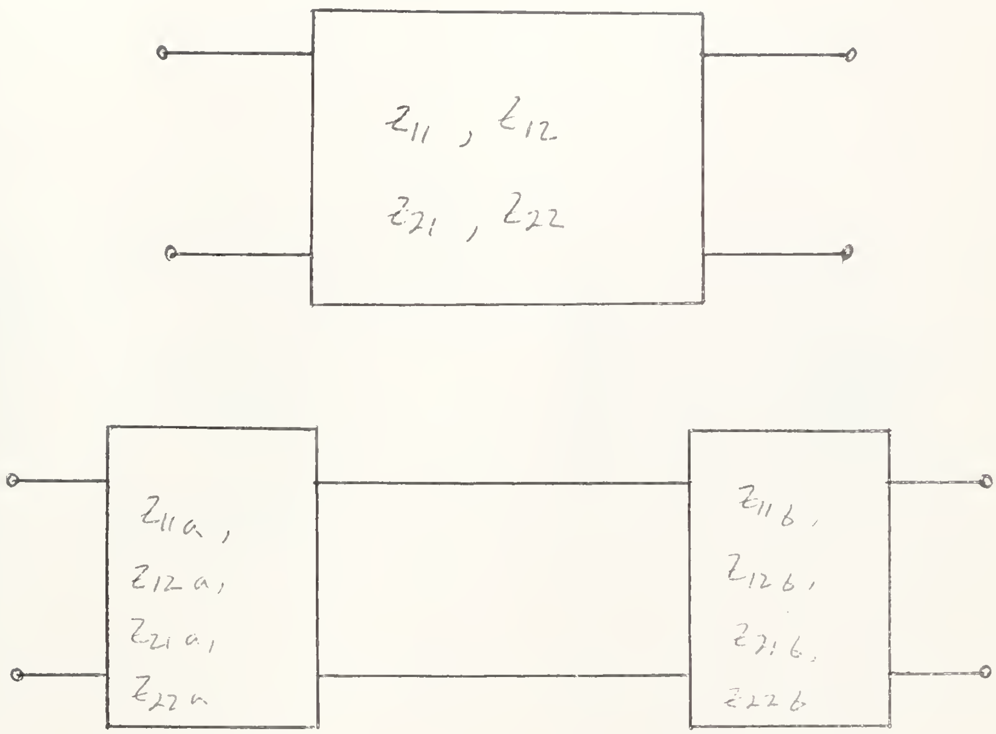


Figure 2

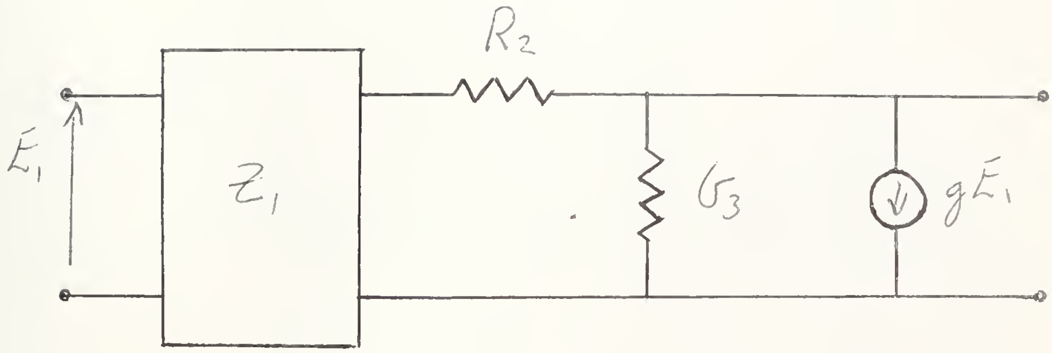


Figure 3

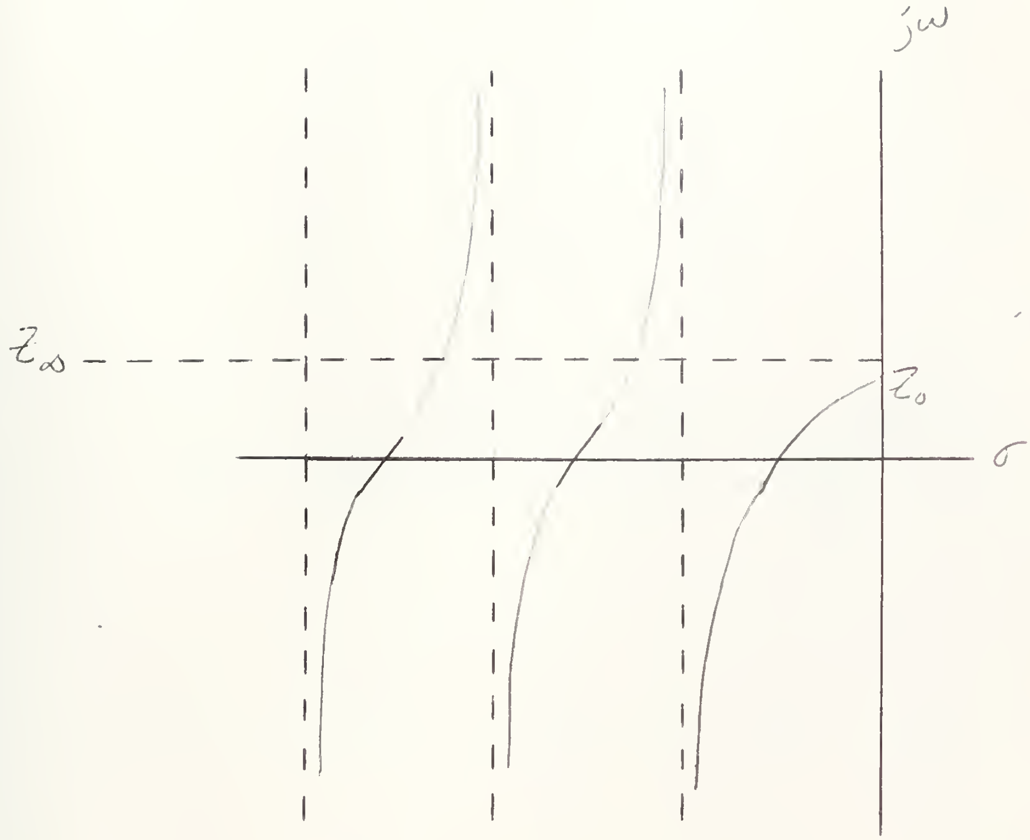


Figure 4

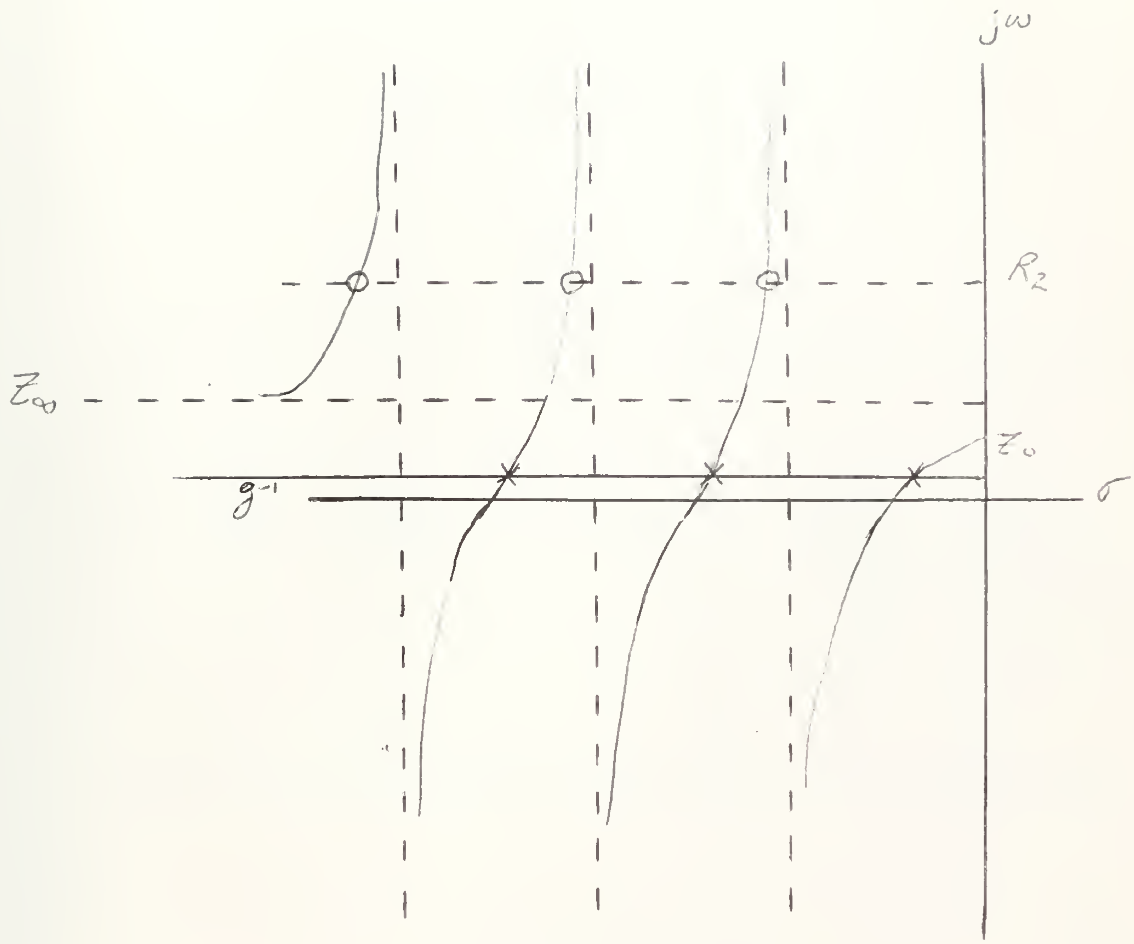


Figure 5

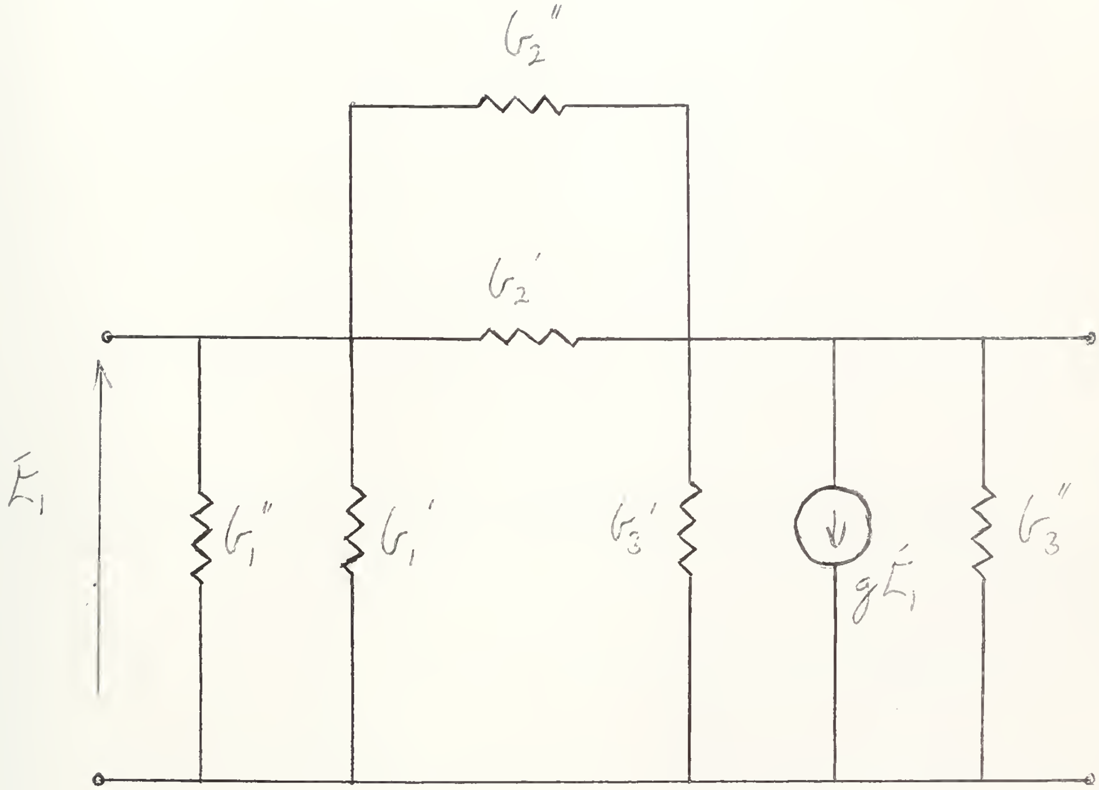


Figure 6

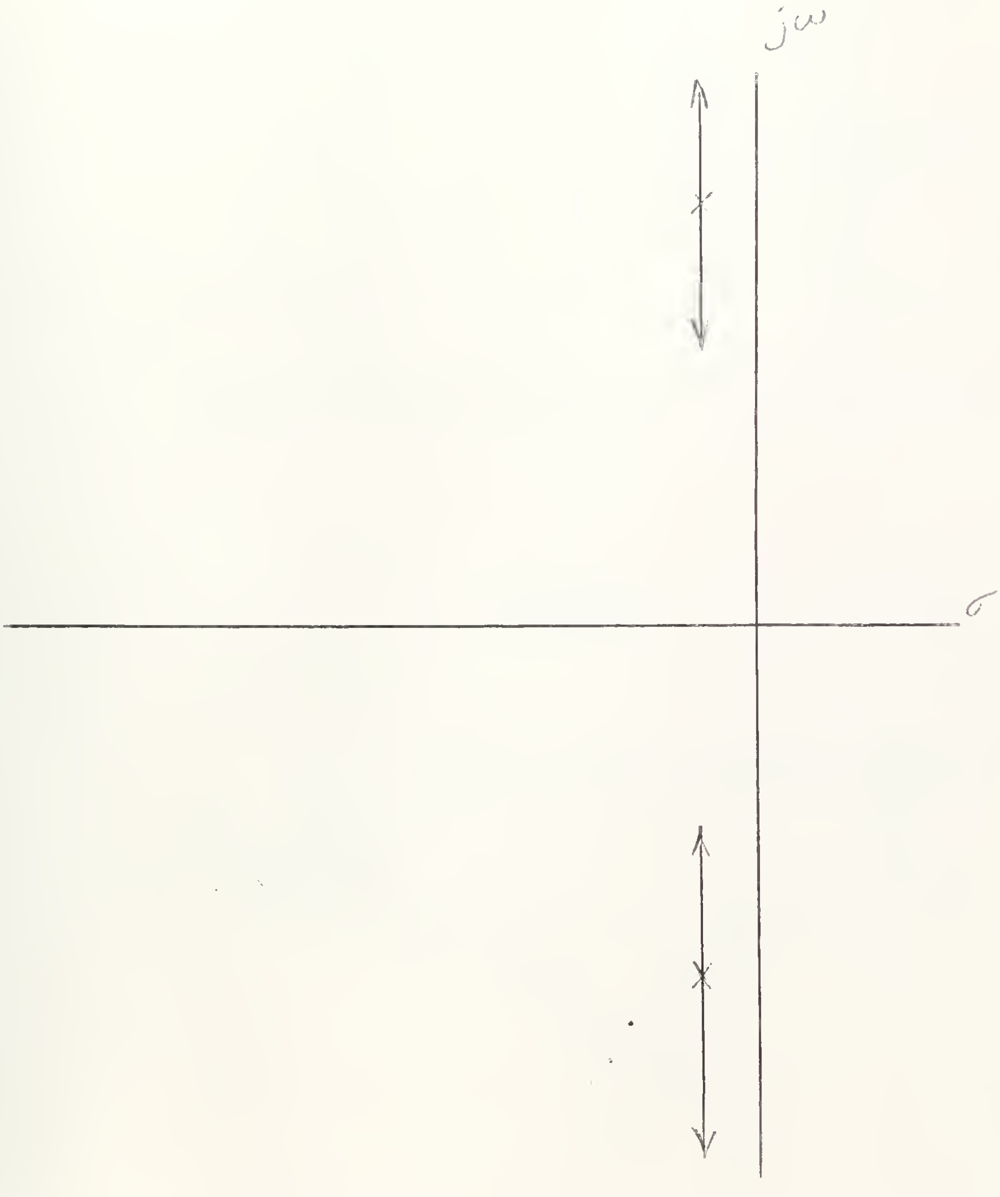


Figure 7

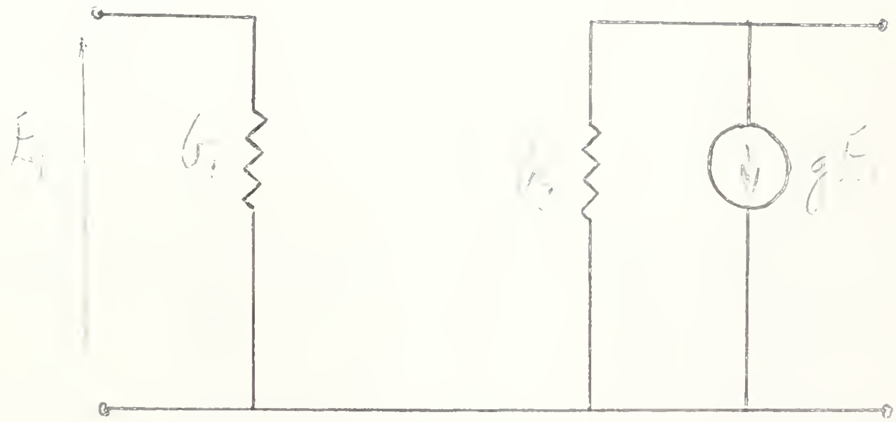
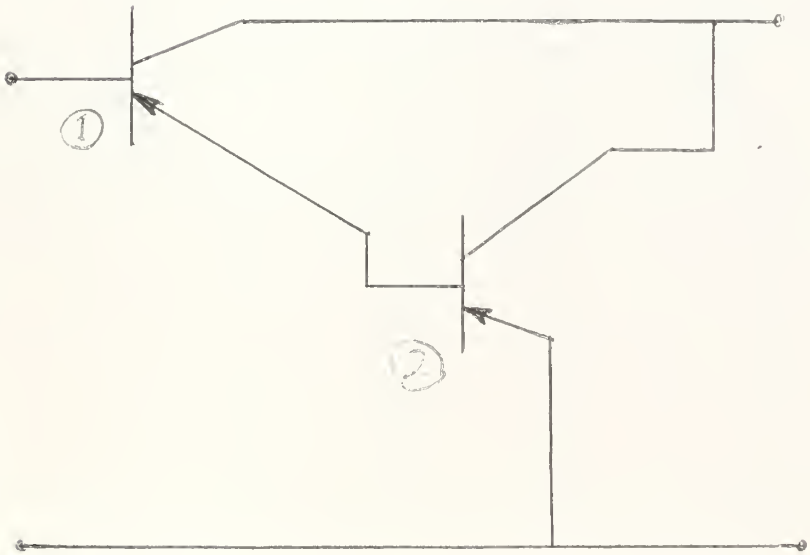


Figure 8



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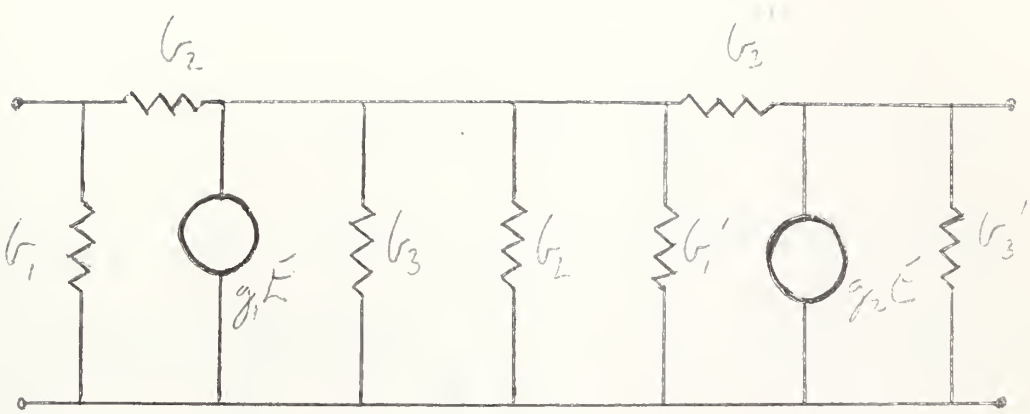

Figure 10

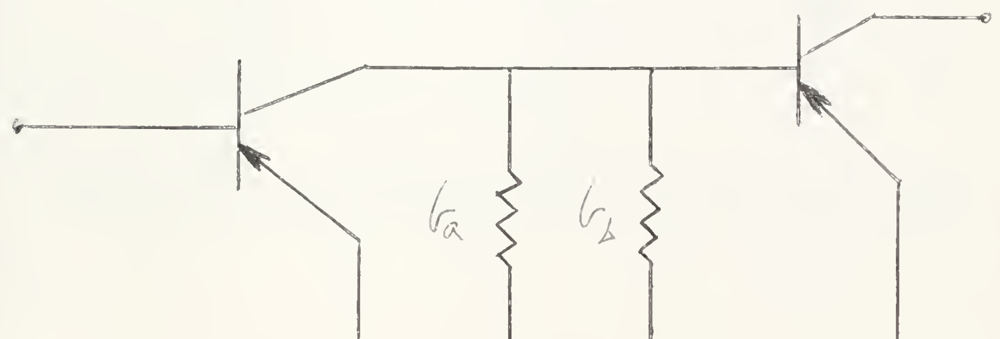

Figure 11

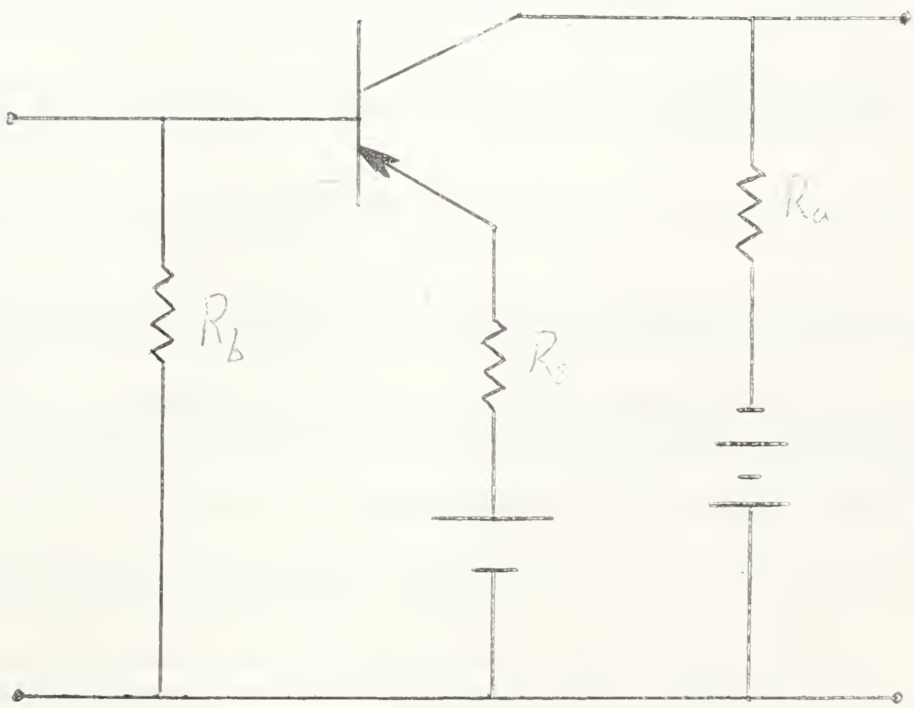

Figure 12

BIBLIOGRAPHY

1. H. H. Scott, A New Type of Selective Circuit and some Applications, Proc. I.R.E., 26, Feb. 1938.

2. J. G. Truxal, Control System Synthesis, McGraw-Hill, pp. 203-218, 1955.

3. J. G. Linvill, RC Active Filters, Proc. I.R.E., 42, pp. 555-564, March 1954.

4. J. G. Linvill, Transistor Negative Impedance Converters, Proc. I.R.E., 41, pp. 725-729, June 1953.

5. I. M. Horowitz, Design of Active RC Filters with Prescribed Insensitivity, manuscript for PGCT, Jan. 1960.

6. I. M. Horowitz, Active RC Transfer Function Synthesis by Means of Cascaded RL and RC Structures, Rome Air Development Center contract AF-30(602)-1648, Feb. 1958.

7. I. M. Horowitz, Active Network Synthesis, I.R.E. National Convention Record Part 2, March 1956.

8. L. Weinberg, Network Synthesis, Control Engineer's Handbook, First Edition, edited by J. G. Truxal, McGraw-Hill, page 4-4, 1958.

9. L. Weinberg, Network Synthesis, Control Engineer's Handbook, First Edition, edited by J. G. Truxal, McGraw-Hill, page 4-5, 1958.

10. J. G. Truxal, Control System Synthesis, McGraw-Hill, page 120.

11. H. W. Bode, Network Analysis and Feedback Amplifier Design, pp. 52-56, D. Van Nostrand Co., 1945.

12. A. R. Pearlman, Some Properties and Circuit Applications of Super-Alpha Composite Transistors, PGED, 2, Jan. 1955.

13. K. W. Cattermole, Transistor Circuits, MacMillan Co., pp 204-205, 1959.

APPENDIX I

$$S_K^{s_0} = \frac{ds_0}{dK/K} \quad \text{for } s_0 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$

k	Real part of	Imaginary part of
σ_1	$-\frac{\zeta\omega_n}{2}$	0
σ_2	$\frac{\zeta\omega_n N_2}{2(N_2+N_3)}$	$-\frac{\omega_n}{2}$
σ_3	$-\frac{\zeta\omega_n N_2}{2(N_2+N_3)}$	0
g	0	$\frac{\omega_n}{2}$
C_1	$\frac{\zeta\omega_n}{2}$	$\frac{\omega_n}{2}$
C_2	$\frac{\zeta\omega_n}{2}$	$\frac{\omega_n}{2}$

APPENDIX II

The relative reduction in sensitivity $S^{K}_{K'}$ for emitter leg loading by a factor m, i.e. $R_e' = m \times r_e'$:

$$S^{g}_{g'} : m$$

$$S^{\sigma_1}_{\sigma_1'} : \frac{1 + \frac{m R_1'}{R_1''}}{1 + \frac{R_1'}{R_1''}} = x_1$$

$$S^{\sigma_3}_{\sigma_3'} : \frac{1 + \frac{m R_3'}{R_3''}}{1 + \frac{R_3'}{R_3''}} = x_3$$

Since ω_2 is constrained by $\omega_2 \doteq \frac{\omega_3 [\omega_1 f^2 - \quad]}{1 - f^2}$ derived from the basic expression between pole position and sensitivity factors:

$$S^{\sigma_2}_{\sigma_2'} : x_1 x_3 \frac{\left[g' f^2 - \left(1 + \frac{R_1'}{R_1''}\right) \right]}{\left[g f^2 - \left(1 + m \frac{R_1'}{R_1''}\right) \right]}$$

APPENDIX III

In terms of r_e, r_b, r_c, and α the equivalent P^i for the common configurations are:

	CE	CB	CC
G_1	$\frac{1-\alpha}{r_e'}$	$\frac{\alpha}{r_e'}$	$\frac{r_e}{r_e' r_c}$
G_2	$\frac{r_e}{r_e' r_c}$	$\frac{r_b}{r_e' r_c}$	$\frac{1-\alpha}{r_e'}$
G_3	$\frac{r_b}{r_c r_e'}$	$\frac{r_e}{r_e' r_c}$	$\frac{\alpha r_c + r_b}{r_c r_e}$
g	$\frac{\alpha}{r_e'}$	$-\frac{\alpha}{r_e'}$	$-\frac{\text{[illegible]}}{r_e'}$

where $r_e' \times r_c \approx |Z|$

APPENDIX IV

For a generator load impedance of 1,000 ohms, output impedance of 50,000 ohms design a filter with a single complex pole pair of $Q = 10$ and $F_c = 150$ cycles. Assume the transistors will vary by 50 percent from their stated values.

$$G_1'' = 10^{-3}\ \mho \qquad G_3'' = 2\times 10^{-5}\ \mho \qquad \Delta K/K = \tfrac{1}{2}$$

$$f = 5\times 10^{-2} \qquad \omega_n = 940\ \text{r.p.s.}$$

From $Q = 10$, $N_1 = 800$. The total transconductance must be 8/10. N_3 then is 40,000. For a pole shift of one percent or less, the equivalent m is 25 x 2 for two active sources, and twice again when the third is added, or $m = 100$.

The P^i elements for a stage with $m = 10$ are:

$$G_1 = 3.15\times 10^{-5} \qquad G_3 = 5\times 10^{-7}$$

$$G_2 = 3.2\times 10^{-7} \qquad g = 2.96\times 10^{-3}$$

Let $G_L = 4 \times 10^{-4}$. g-cascade in numbers is:

$$g_{CASCADE} = \frac{(2.96)^3 \times 10^{-8}}{16 \times 10^{-8}} = .93$$

This is to be reduced to 8/10, and a 10 ohm resistor is added to the first stage emitter as an additional stabilizer.

To determine the bridge shunt resistor for G_2, N_2 is fixed by:

$$\frac{N_1 N_3}{N_2} = 800$$

or, $N_2 \approx 40{,}000.$ $N_2' = \dfrac{g}{G_2' + G_2''}$ and when stabilized:

$$N_2 = \frac{g}{\frac{G_2}{m}} = \frac{g}{\frac{1}{m}\left[G_2' + G_2''\right]} = \frac{g}{G_2' + \frac{G_2''}{m}}$$

G_2'' is decreased by m while $G_2' \approx \dfrac{R_S}{r_C R_2'}$ remains essentially constant since loading affects the numerator and the denominator.

$$R_2'' = \frac{100}{2 \times 10^{-5}} = 5 \times 10^{6}\ \Omega$$

The values of the capacitors are calculated from (5):

$$g = \frac{f\omega_n N_2 N_3 H}{N_2 + N_3} \qquad C_2 = H \text{ farads}$$

$$C_1 = \frac{f^2 N_2 N_3^2 (N_2 - 1) H}{N_1 \left[f^2 N_2 (1 + N_3) - (N_2 + N_3)\right](N_2 + N_3)}$$

Since N_2 is equal to N_3 and all are large with respect to one, the capacitor values are:

$$C_2 = \frac{2g}{f\omega_n N_2} \qquad C_1 = \frac{C_2 N_3}{2N_1}$$

In numbers:

$$C_1 = 21.3\ \mu f \qquad C_2 = .85\ \mu f$$

For a one percent or less shift in frequency, the precise elements, C_1, C_2, and R_2'' must have tolerances of two percent or less.

The final circuit is illustrated in Figure 13.

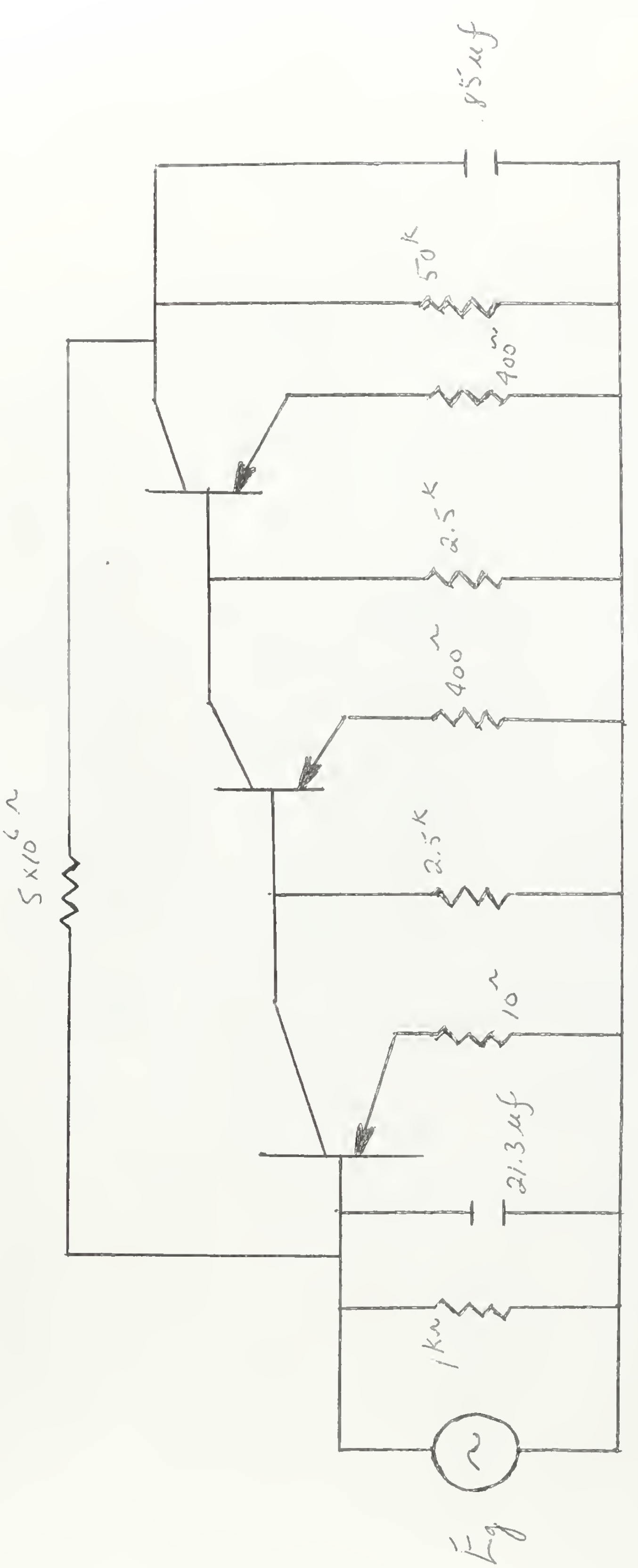

Figure 13